Theory

for

CT SAT Calculator (PSRC)

Original document: 4 June 2001. 30 Sept. 03: changed cos to sin in (8), (9), (10); *RP to 1/RP in (12) and (13)*

Spreadsheet originated by the IEEE PSRC committee responsible for

C37.110 "IEEE Guide for the Application of Current Transformers Used for Protective Relaying Purposes"

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Introduction

The spread sheet "CT Saturation Calculator" is intended to provide quick indication not only of whether or not a CT will saturate in a particular application, but also an accurate indication of the actual waveshape of the secondary current so that the degree of saturation as a function of time is apparent. Furthermore, the data is available to the user to use as input to a digital relay model, if such is available. The user can convert the data into a COMTRADE file, for example.

There are many technical papers on the subject of modeling the behavior of iron-cored current transformers used for protective relaying purposes. One of the difficulties in using an *elaborate* model (in any field of engineering) is in getting the parameters in a particular case in order to implement that model easily, efficiently and accurately. For example, the excitation current in the region below the knee-point is a complex combination of magnetizing, hysteresis and eddy-current components, the parameters of which are usually not known in a particular case.

It turns out (can be shown) that, if the excitation current waveform reaches into the saturated region, the part of the waveform in the below-knee-point region has negligible effect on the overall solution. This simplifies the solution greatly, with little effect on accuracy.

If errors under low current, low burden conditions are of interest, a more elaborate model must be used.

Testing of the Model

The proof of the pudding is in the eating. Because this model is new and quite different from those in the literature, testing against real high-current laboratory results was important. To this end, two laboratory examples published in reference (1) were compared against results from this program. The agreement was very close. [Note the comment at the end of reference (1).]

In addition, the program has had widespread circulation, and to date there are two utility-user reports of agreement with previous results and no reports of disagreement.

Circuit model

The circuit model is shown in Fig. 1.

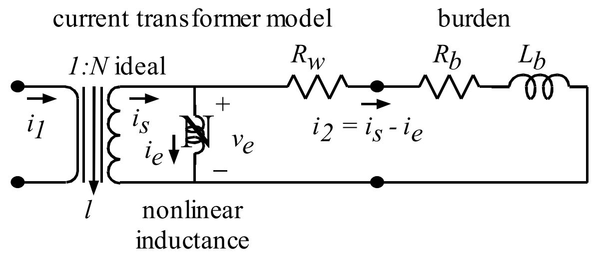

Fig. 1 Circuit model.

The symbols used in Fig.1 as well as those used in what follows and on the spread sheet are listed in the next section.

Symbols

All units are SI: volts, amps, weber-turns, ohms, henries, radians, seconds.

i_1	instantaneous primary current
I_P	rms symmetrical primary fault current
Off	dc-offset magnitude (per unit)
τ_1	system time constant
i_2	instantaneous secondary current
i_s	instantaneous ideal secondary current
i_e	instantaneous excitation current
I_e	rms excitation current
I_{pk}	peak excitation current
v_e	instantaneous excitation voltage
V_e	rms excitation voltage
V_s	rms 'saturation voltage'
λ	instantaneous flux-linkages
λ_{rem}	remanence (per unit of V_s)
S	inverse of slope of V_e vs I_e curve
A	parameter of i_e vs λ curve
RP	factor defined as I_e / I_{pk}
ω	radian frequency = $2\pi 60$
T	one period: 2π radians
R_w	winding resistance
R_b	burden resistance
R_t	$R_w + R_b$
L_b	burden inductance

The Excitation Curve

The excitation characteristic of the CT is invariably a plot of secondary rms voltage versus secondary rms current, on log-log axes, as shown in Fig. 2.

For this model, only two parameters need to be extracted from the curve: S and V_S . See Fig. 3.

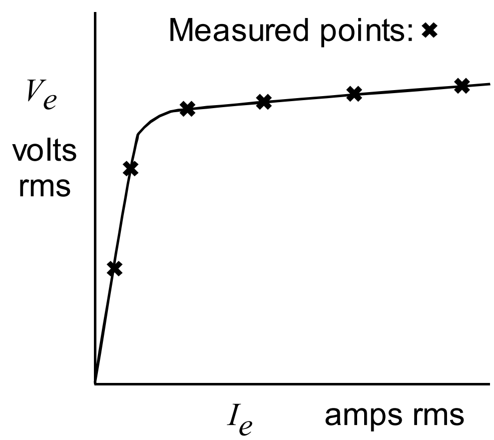

Fig. 2 Factory-supplied information: the excitation curve.

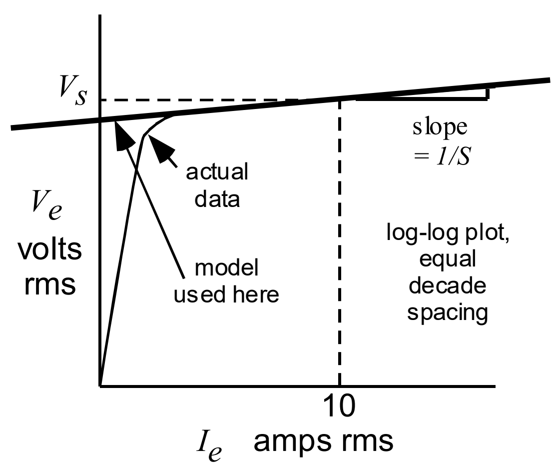

Fig. 3 Method of determining the parameters V_s and S
for the saturation curve used in the model.

The reason for choosing the *saturation voltage,* V_s , at the point where the excitation current is ten amps, is that this is the definition used in the standard. For example, a C400 CT is one in which the excitation voltage is 400 volts rms (or more) for an error current of 10 amps. Caution: in setting up a particular case, use the *actual* value for V_s rather than the *rating* value because a CT rated C400 may actually supply, for example, 423 volts at 10 amps.

In order to check the validity of ignoring the high-slope low-end of the saturation curve, two models were compared: one the model of Fig. 3, and another the model of Fig. 4. As long as the condition was at or near saturation, there was no visible difference in the saturation curves, because the below-the-knee-point currents are very small by comparison with even mild saturation currents. The decided advantage of eliminating this region from the model is that the hysteresis and eddy current parameters are very difficult to determine. They are not included in standard data for CT's.

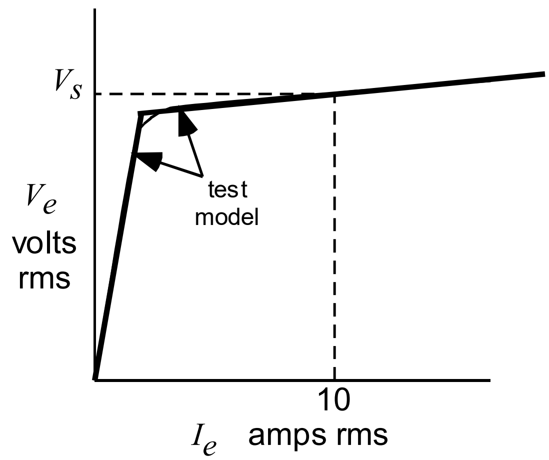

Fig. 4 Temporary test model.

Conversion to Instantaneous Quantities

The straight line curve with slope $1/S$ shown in Fig. 3 is <u>not</u> linear. It is a curve defined mathematically as

$$\log V_e = \frac{1}{S}\log I_e + \log V_i \tag{1}$$

where V_i is the value of V_e for I_e=1, that is for log I_e=0. Removing the logs:

$$V_e = V_i I_e^{1/S}. \tag{2}$$

Remember that these are all *rms* quantities, presumably measured with "true-rms" voltmeters and ammeters. [A study has shown that if rms-calibrated meters were used, with either peak-sensitive or rectified-average-sensitive elements, the effect on accuracy is not substantial.]

In order to solve the differential equations implied by the circuit of Fig. 1, one needs the instantaneous λ versus i_e curve. It is postulated here that a curve defined as

$$i_e = A \cdot \lambda^S \tag{3}$$

is suitable as long as the exponent S is an odd integer. In order to allow S to be <u>any</u> positive number, and keep the function odd, we can use the following more general expression:

$$i_e = A \cdot \mathrm{sgn}(\lambda) \cdot |\lambda|^S \tag{4}$$

where $\mathrm{sgn}(\lambda)$ is the sign of λ. See Fig. 5 showing a sample plot of this function.

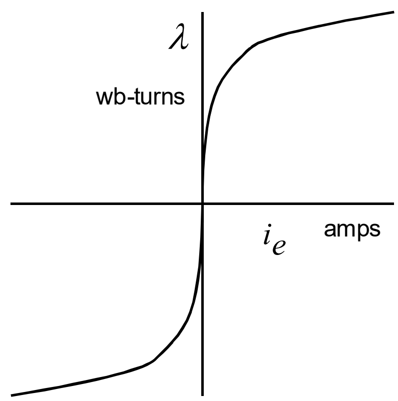

Fig. 5. Postulated instantaneous values saturation curve.

The next step is to determine the constant A in terms of known parameters.

First, the flux-linkages λ are related to the instantaneous excitation voltage v_e by Faraday's law (The error due to ignoring R_w here is very small - less than the measurement errors involved in determining the V_e vs I_e curve):

$$v_e = \frac{d\lambda}{dt} \tag{5}$$

The excitation curve is found using sinusoidal voltage, which implies that the flux-linkages are also sinusoidal:

$$v_e = \sqrt{2}V_e \cos(\omega t) \text{, and} \quad (6)$$

$$\lambda = \int v_e dt = \int \sqrt{2}V_e \cos(\omega t)dt = \sqrt{2}V_e \frac{1}{\omega}\sin(\omega t) . \quad (7)$$

The excitation current is non-sinusoidal, since it is an S^{th} order function of λ:

$$i_e = A\lambda^S = A\left[\frac{\sqrt{2}V_e}{\omega}\sin(\omega t)\right]^S = A\left[\frac{\sqrt{2}V_e}{\omega}\right]^S \sin^S(\omega t) \quad (8)$$

The rms value of this current is, by definition:

$$I_e = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i_e^2 dt} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} A^2\left[\frac{\sqrt{2}V_e}{\omega}\right]^{2S} \sin^{2S}(\omega t)dt}$$

$$= A\left[\frac{\sqrt{2}V_e}{\omega}\right]^S \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \sin^{2S}(\omega t)dt} \quad (9)$$

Next, we define the ratio of rms-value-to-peak-value of the excitation current as RP :

$$RP = \frac{rms}{peak} .$$

For a sinusoid RF=0.7071, and for i_e RP is given by

$$RP = \frac{\sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_{pk}^2 \sin^{2S}(\omega t)dt}}{I_{pk}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \sin^{2S}(\omega t)dt} \quad (10)$$

The above definite integral, (and hence RP) can best be evaluated using numerical integration, and in fact this is done directly on the spread sheet - using trapezoidal integration - for the particular S entered by the user.

Fig. 6 illustrates the difference between rms/peak for a sinusoid and rms/peak for the assumed excitation current waveform: the form factor RP gets smaller as the value of S increases.

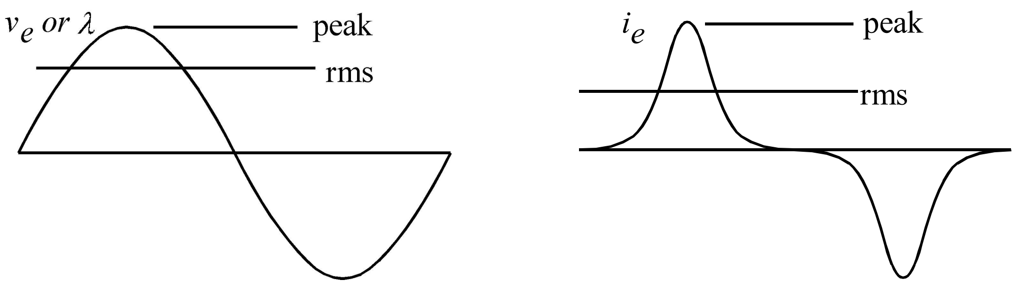

Fig.6 Comparison of the rms/peak relationship for two waveshapes.
Left: excitation voltage or flux-linkages. Right: excitation current.

Substituting this result into equation (8), yields

$$I_e = A\left[\frac{\sqrt{2}V_e}{\omega}\right]^S RP \tag{11}$$

But we know that when I_e=10 , V_e=V_s. Substituting,

$$10 = A\left[\frac{\sqrt{2}V_s}{\omega}\right]^S RP\,.$$

Solving for A:

$$A = \frac{10\omega^S}{\left(\sqrt{2}V_s\right)^S}\frac{1}{RP}. \tag{12}$$

Equation (4) becomes, therefore, the fundamental i_e *vs* λ relationship:

$$i_e = \mathrm{sgn}(\lambda)\frac{10\omega^S}{\left(\sqrt{2}V_s\right)^S}\frac{1}{RP}\cdot|\,\lambda\,|^S \quad \text{as illustrated in Fig. 5.} \tag{13}$$

Solution of Circuit Model

The circuit of Fig. 1 is solved simply by writing Kirchhoff's Voltage Law around the right-hand loop:

$$v_e - (i_s - i_e) \cdot R_t - L_b \frac{d}{dt}\left[i_s - i_e\right] = 0 \tag{14}$$

The 'forcing function' and its derivative are:

$$i_s = \frac{i_1}{N} = \frac{\sqrt{2} \cdot I_p}{N}\left[Off \cdot e^{-t/Tau1} - \cos(\omega t - \cos^{-1} Off)\right] \tag{15}$$

$$\frac{di_s}{dt} = \frac{\sqrt{2} \cdot I_p}{N}\left[\frac{-Off}{Tau1} \cdot e^{-t/Tau1} + \omega \cdot \sin(\omega t - \cos^{-1} Off)\right] \tag{16}$$

Note that

$$\frac{di_e}{dt} = \frac{di_e}{d\lambda} \cdot \frac{d\lambda}{dt} \tag{17}$$

and

$$\frac{di_e}{d\lambda} = A \cdot S \cdot |\lambda|^{S-1}. \tag{18}$$

Finally, with substitutions and manipulation, equation (14) is re-written as:

$$\underbrace{\frac{d\lambda}{dt} \cdot \left[1 + L_b \cdot A \cdot S \cdot |\lambda|^{S-1}\right] = -R_t i_e}_{\text{dependent variable}} + \underbrace{R_t i_s + L_b \frac{di_s}{dt}}_{\text{forcing function}} \tag{18}$$

This first-order nonlinear differential equation is solved for $\lambda(t)$ using standard numerical analysis techniques, such as trapezoidal integration, Runge-Kutta integration, or simple step increments. The latter is used in the spread sheet program, for simplicity, since the accuracy is sufficient for this application.

Then the excitation(error) current i_e is given by equation (3), and the actual secondary current - the goal of this exercise - by

$$i_2 = i_s - i_e \tag{19}$$

Remanence

With the single-valued saturation curve assumed here, conventional remanence is not possible because non-zero λ cannot occur for zero i_e. However, remanence can be approximated very closely by simply assuming that the initial excitation current is non-zero. A quite small initial excitation current will accomplish this, even for a large remanence. For convenience, λ_{rem} is expressed in per unit of V_s since the "knee-point" itself is not defined in this model. See Fig. 7. In order to specify λ_{rem} accurately, x must be specified no greater than V_{knee} in Fig. 7. In other words, if V_{knee} is 80% of V_S then the value of λ_{rem} cannot exceed 0.8 per unit.

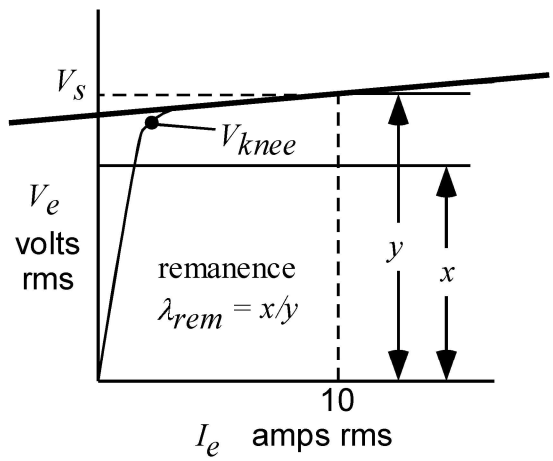

Fig. 7 Definition of per unit remanence used in this model.

References

(1) Tziouvaras, D.A., et al, Mathematical Models for Current, Voltage, and Coupling Capacitor Voltage Transformers, Working Group C5 of PSRC, IEEE Trans. on Power Delivery, Jan 2000, pp. 62-72. (Note that there is an error in figures 4a and 5a of this paper: there was actually non-zero remanence for this case, as confirmed with the authors.)